

## LOCI 4024

COMPILED BY : MUSTAFA ASIF

### **33. Loci**

- use the following loci and the method of intersecting loci for sets of points in two dimensions which are:
  - (a) at a given distance from a given point
  - (b) at a given distance from a given straight line
  - (c) equidistant from two given points
  - (d) equidistant from two given intersecting straight lines

For clearing concepts

<https://www.youtube.com/watch?v=dtmMOF07kgE>

<https://www.youtube.com/watch?v=wDJrOWMeYOc>

<https://www.youtube.com/watch?v=Elk2C8NQs0Y>

## Locus and Constructions

### Key Points

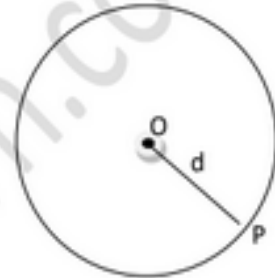
#### Locus:

In geometry, a **locus** (Latin for "place", plural *loci*) is a collection of points which share a property. For example, a circle may be defined as the locus of points in a plane at a fixed distance from a given point. A locus may alternatively be described as the path through which a point moves to fulfill a given condition or conditions. For example, a circle may also be defined as the locus of a point moving so as to remain at a given distance from a fixed point.

#### Loci in Two Dimensions

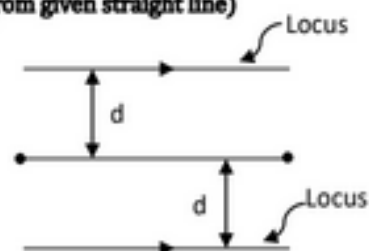
➤ **Circle: (Locus of a point at given distance from a fixed point)**

In general, the locus of a point P which is at a given distance  $d$  from a given point O is a circle with centre O and radius  $d$ .



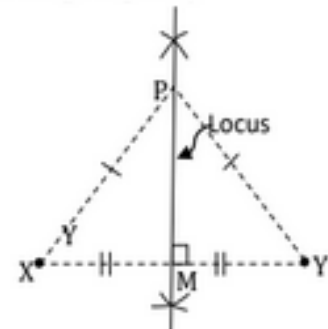
➤ **Parallel Lines: (Locus of a point at given distance from given straight line)**

In general, the loci of point which is at a given distance  $d$  from a given straight line XY are two straight lines parallel to XY and at a distance  $d$  from XY.



➤ **Perpendicular Bisector Of Line: (Locus of a point equidistant from two given points)**

In general, the locus of a point which is equidistant from two given points X and Y is the perpendicular bisector of the line XY.



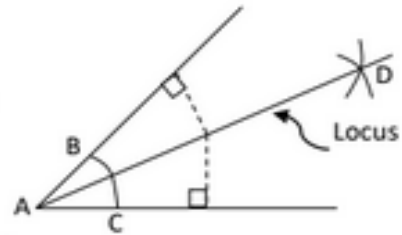
#### How to draw Perpendicular Bisector

- Place the compass at one end of line.
- Adjust the compass to slightly longer than half the line length
- Draw arcs above and below the line.
- Keeping the same compass width, draw arcs from other end of line.
- Place ruler where the arcs cross, and draw the line.

In general, the locus of a point which is equidistant

From two given intersecting straight lines is a straight

Line which bisects the angle between the two given lines.



**How to draw Angle Bisector:**

- Place the compass on the angle point A.
- Draw an arc BC which intersects both arms of angle.
- Place compass on point B and draw an arc.
- Place compass (compass opening must not be changed) on point C and draw one more arc.
- Both arcs will intersect at point D.
- Draw a straight line by joining point A and point D. The line AD will be the angle bisector.

➤ **Intersections of Loci :**

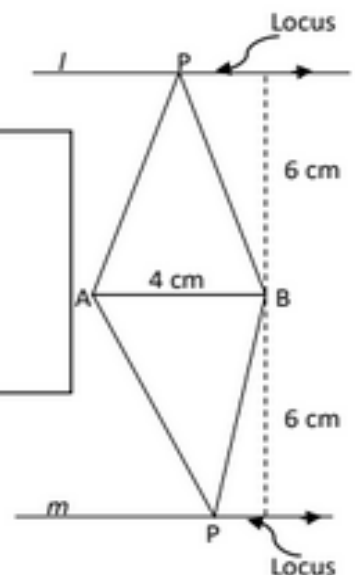
If two or more loci intersect at a point P, then P satisfies the conditions of the loci simultaneously.

➤ **Locus of a point for which area of given triangle remain constant.**

The locus of a point P which moves such that the area of  $\triangle ABP$  remains constant is a set of points of two lines  $l$  and  $m$  parallel to and equidistant from  $AB$ .

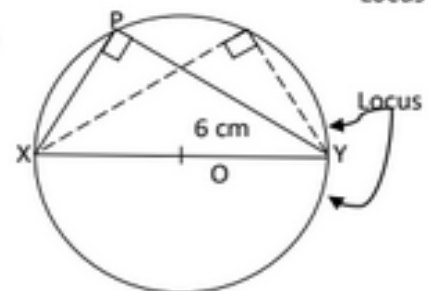
If  $AB = 4 \text{ cm}$  and area of triangle is  $12 \text{ cm}^2$ , then the locus of point P is shown in figure.

Area of  $\triangle ABP = 12 \text{ cm}^2$   
 Area =  $\frac{1}{2} B \times h$   
 $12 = \frac{1}{2} \times 4 \times h$   
 $h = 6 \text{ cm}$



➤ **Locus of points at which a line segment subtends an angle of  $90^\circ$**

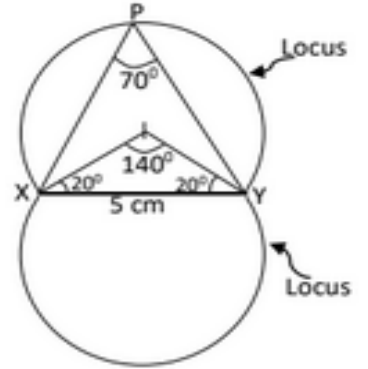
Give that a line  $XY = 6 \text{ cm}$  such that  $\angle XPY = 90^\circ$ , then the locus of P is the set of points excluding X and Y, on the circumference of the circle with XY as diameter.



➤ **Locus of points at which a line segment subtends a given angle**

Given that a line  $XY$  5 cm,  $\angle XPY = 70^\circ$ , the locus of the point  $P$  the set of the points on the arc  $XPY$  and its reflection in  $XY$ , excluding  $X$  and  $Y$ ,

$\angle XOY = 2 \times 70^\circ = 140^\circ$  and  $\angle XPO = 20^\circ$



**Shading of Loci**

Sr.	Condition	Shading
1	$AP \geq PB$	
2	$AP \leq PB$	
3	$AP > PB$	
4	$AP < PB$	

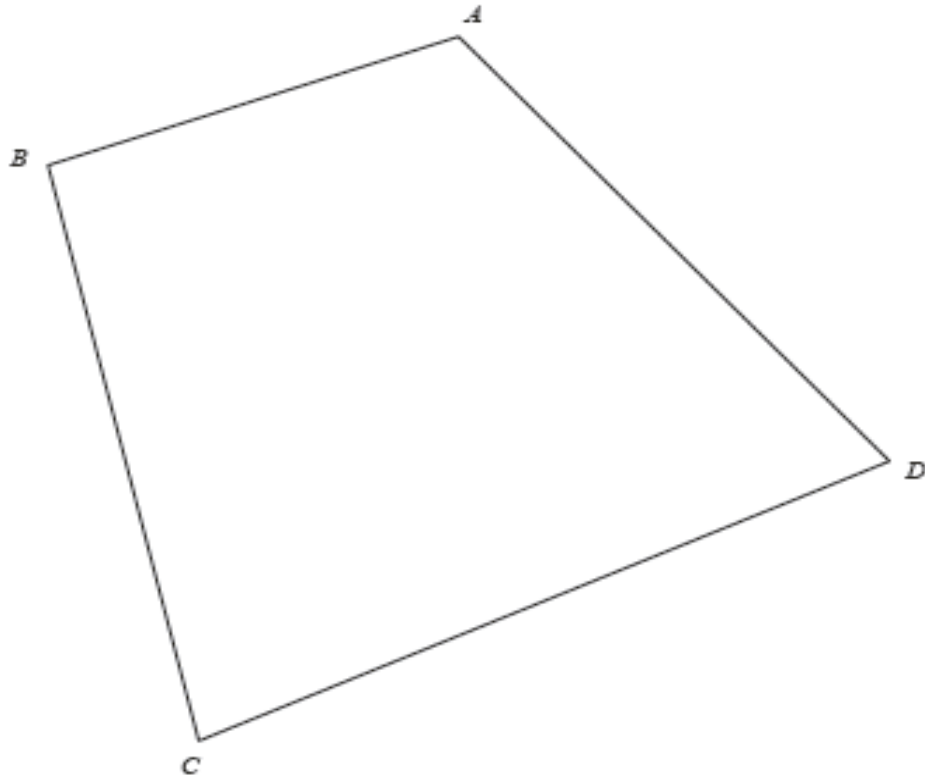
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5	$\widehat{BAQ} \leq \widehat{CAQ}$	
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Sr.	Condition	Shading
6	$OP \leq 2 \text{ cm}$	
7	$OP < 2 \text{ cm}$	
8	$1 \text{ cm} \leq OP \leq 2 \text{ cm}$	
9	$OP \geq 2 \text{ cm}$	
10	$OP > 2 \text{ cm}$	

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1 The diagram shows the plan of a garden,  $ABCD$ .

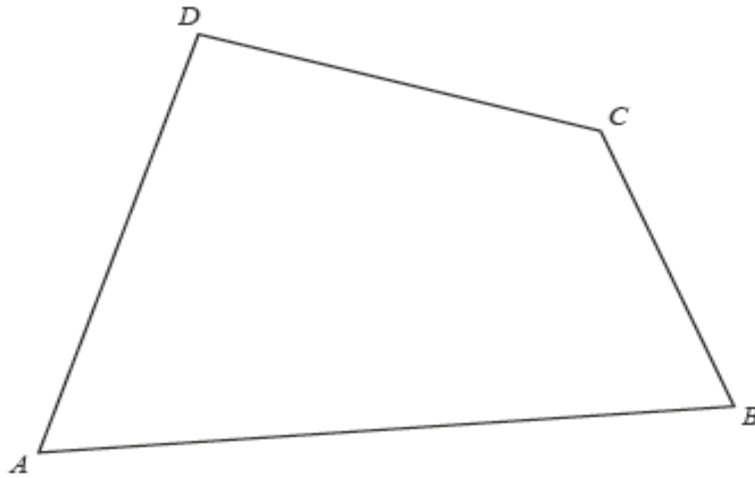


- (a) Use a straight edge and compasses only to construct the locus of points inside  $ABCD$  that are
- (i) equidistant from  $A$  and  $B$ , [2]
  - (ii) equidistant from  $CD$  and  $AD$ . [2]
- (b) Stefan wants to plant a tree in the garden.  
He wants the tree to be
- nearer to  $A$  than to  $B$   
and
  - nearer to  $CD$  than to  $AD$ .

Shade the region inside the garden where Stefan can plant the tree. M/J19/11/14 [1]

- 2 Use a straight edge and compasses only in this question.

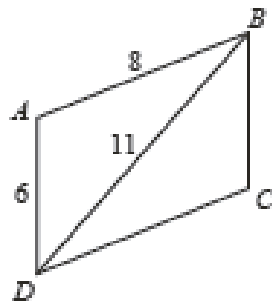
Construct the locus of points inside quadrilateral  $ABCD$  that are equidistant from  $AB$  and  $BC$ .



[2]

M/J19/12/2

3 (a)



NOT TO  
SCALE

M/J19/21/5(a)

*ABCD* is a parallelogram.  
*AD* = 6 cm, *AB* = 8 cm and *BD* = 11 cm.

- (i) Using a ruler and compasses only, construct an accurate drawing of *ABCD*.  
*AD* has been drawn for you.



[3]

- (ii) Measure  $\hat{DAB}$ .

$\hat{DAB} = \dots\dots\dots$  [1]

- (iii) *E* is the point on *BD* such that *AE* is the shortest distance from *A* to *BD*.

Draw and measure *AE*.

*AE* =  $\dots\dots\dots$  cm [1]



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4 The diagram at the bottom of the page shows the lines  $AB$  and  $BC$ .

(a) By measuring an angle, find reflex angle  $ABC$ .

*Answer*  $\hat{ABC} = \dots\dots\dots$  [1]

(b) The point  $D$  is on the opposite side of  $AC$  to  $B$ .      SP18/01/22  
 $CD = CB$  and  $AD = 10$  cm.

On the diagram, construct quadrilateral  $ABCD$ . [1]

(c) On the diagram, construct the locus of points, inside the quadrilateral  $ABCD$ , that are

(i) equidistant from  $A$  and  $B$ . [1]

(ii) equidistant from  $BC$  and  $BA$ . [1]

(d) On the diagram, shade the region inside the quadrilateral  $ABCD$  containing the points that are

- nearer to  $A$  than to  $B$
- and nearer to  $BC$  than to  $BA$ .

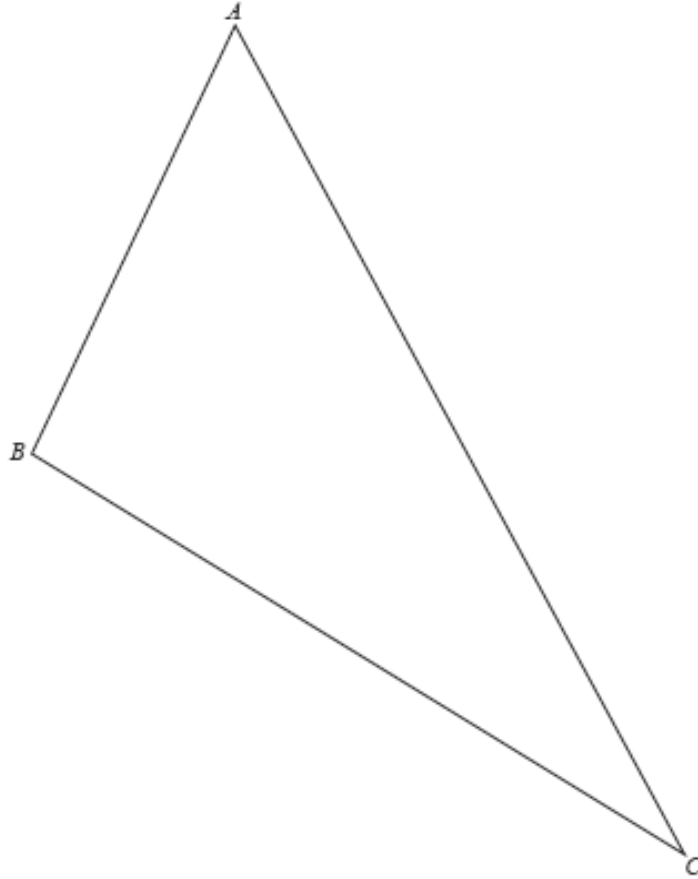


[1]

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5

O/N18/11/19



(a) Measure angle  $ABC$ .

*Answer* ..... [1]

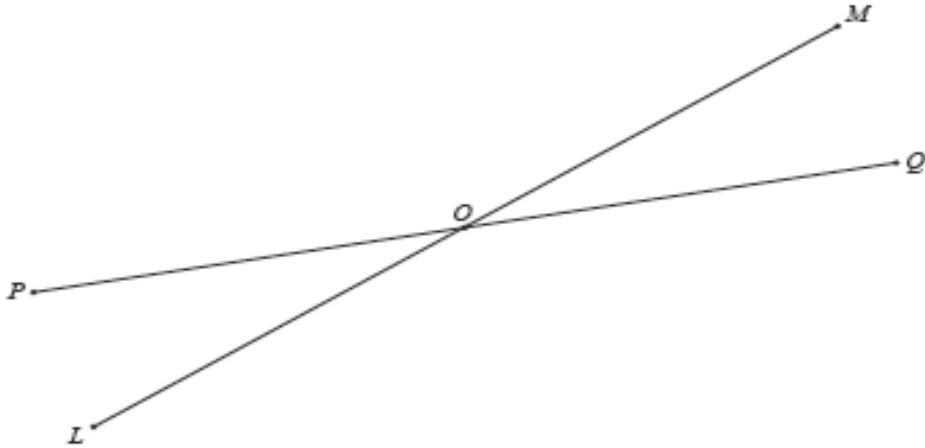
(b) In this part, use a pair of compasses and a straight edge only.

(i) Construct the bisector of angle  $BAC$ . [2]

(ii) Construct the perpendicular bisector of  $AB$ . [2]

6 In the diagram,  $PQ$  intersects  $LM$  at  $O$ .

O/N18/12/19



On the diagram, construct

(a) the locus of points that are equidistant from the lines  $PQ$  and  $LM$ ,

[2]

(b) the locus of points that are equidistant from  $O$  and  $Q$ .

[1]

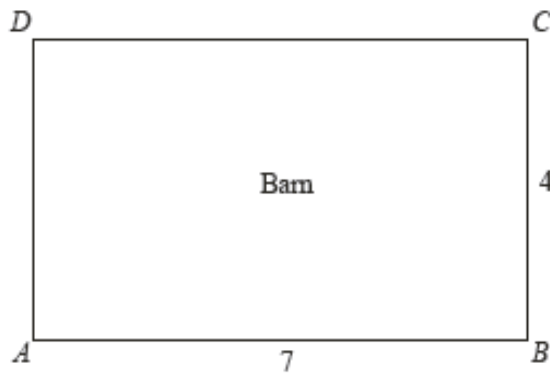
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- 7 The scale diagram below shows a barn  $ABCD$ . M/J18/11/10  
 $AB = 7\text{ m}$  and  $BC = 4\text{ m}$ .  
On the diagram 1 cm represents 1 m.

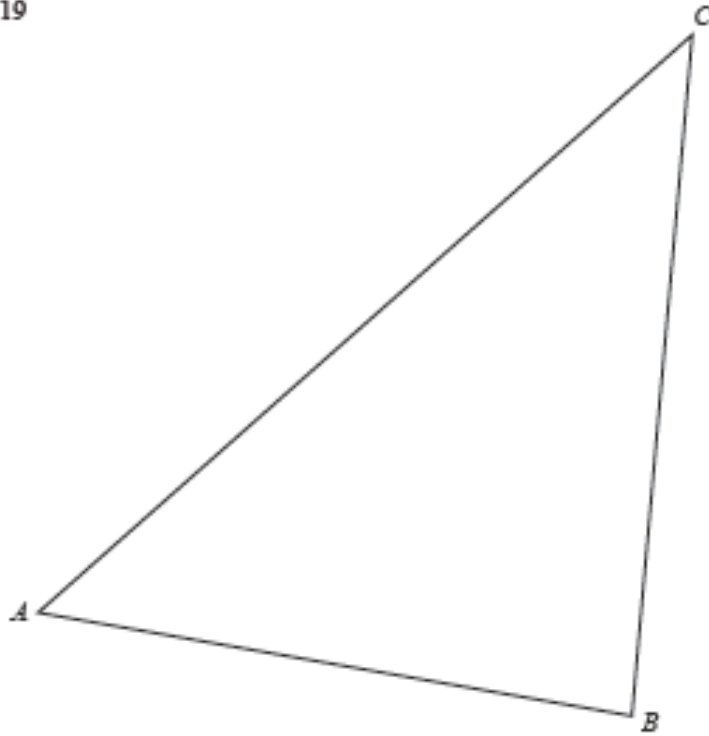
A horizontal rail is attached to the outside wall of the barn from  $A$  to  $B$ .  
Jasper is a dog attached to a rope 3 m long.  
The other end of the rope is attached to the rail and can slide along it.

On the diagram, shade the region where Jasper can go.

[3]



Scale: 1 cm to 1 m.

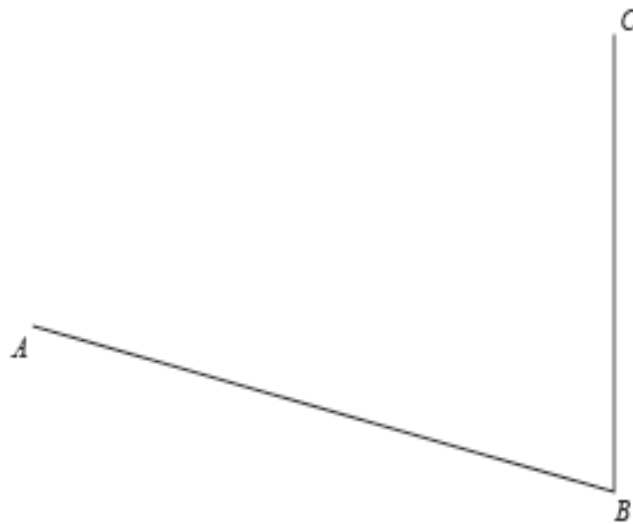


- (a) On the diagram, construct the perpendicular bisector of  $AB$ . [1]
- (b) On the diagram, construct the locus of points inside triangle  $ABC$ , that are
- (i) 7 cm from  $C$ , [1]
  - (ii) equidistant from  $AB$  and  $AC$ . [1]
- (c)  $P$  is any point which is
- equidistant from  $A$  and  $B$   
and more than 7 cm from  $C$   
and nearer to  $AC$  than  $AB$ .
- Find the extremes of the possible positions of  $P$  and label them  $P_1$  and  $P_2$ . [1]

- 9 (a)  $ABC$  is a triangle with  $AC = 6$  cm and  $BC = 9$  cm.  
 $AB$  has been drawn below. M/J18/21/6(a)



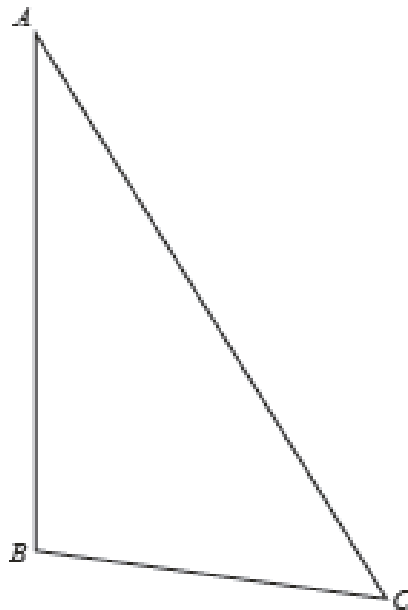
- (i) Using a ruler and a pair of compasses only, construct triangle  $ABC$ . [2]
- (ii) Measure  $\hat{BAC}$ .



The diagram shows the lines  $AB$  and  $BC$ .

The point  $D$  is on the opposite side of  $AC$  to  $B$ .  
 $AD = 5$  cm and  $CD = 6.5$  cm.

- (a) Construct quadrilateral  $ABCD$ . [1]
- (b) On the diagram, construct the locus of points, **inside** the quadrilateral, that are
- (i) equidistant from  $AB$  and  $BC$ , [1]
  - (ii) equidistant from  $B$  and  $C$ . [1]
- (c) The line  $PQ$  consists of the points, inside the quadrilateral, which are
- equidistant from  $AB$  and  $BC$ ,  
and nearer to  $C$  than to  $B$ .
- Mark and label the line  $PQ$  on the diagram. [1]



The diagram shows the triangle  $ABC$ .

- (a) Measure angle  $ABC$ .

*Answer* ..... [1]

- (b) On the diagram, construct the perpendicular bisector of  $AB$ . [1]

- (c) On the diagram, construct the locus of points that are 5 cm from  $C$ . [1]

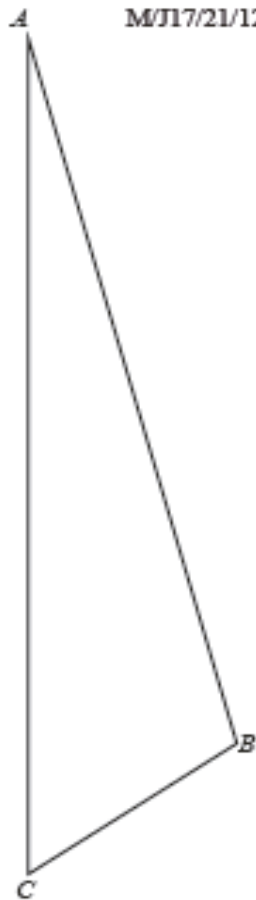
- (d) The points  $P$  and  $Q$  lie on the perpendicular bisector of  $AB$  and are 5 cm from  $C$ .

Mark and label the points  $P$  and  $Q$  on the diagram and measure  $PQ$ .

*Answer*  $PQ =$  ..... cm [1]



12 (a)



$ABC$  is a triangle.  
 $B$  and  $D$  are points on opposite sides of the line  $AC$ .  
 $DA = 9$  cm and  $CD = 7$  cm.

(i) Accurately draw and label the point  $D$ . [2]

(ii) Measure  $\hat{DAB}$ .

Answer  $\hat{DAB} = \dots\dots\dots$  [1]

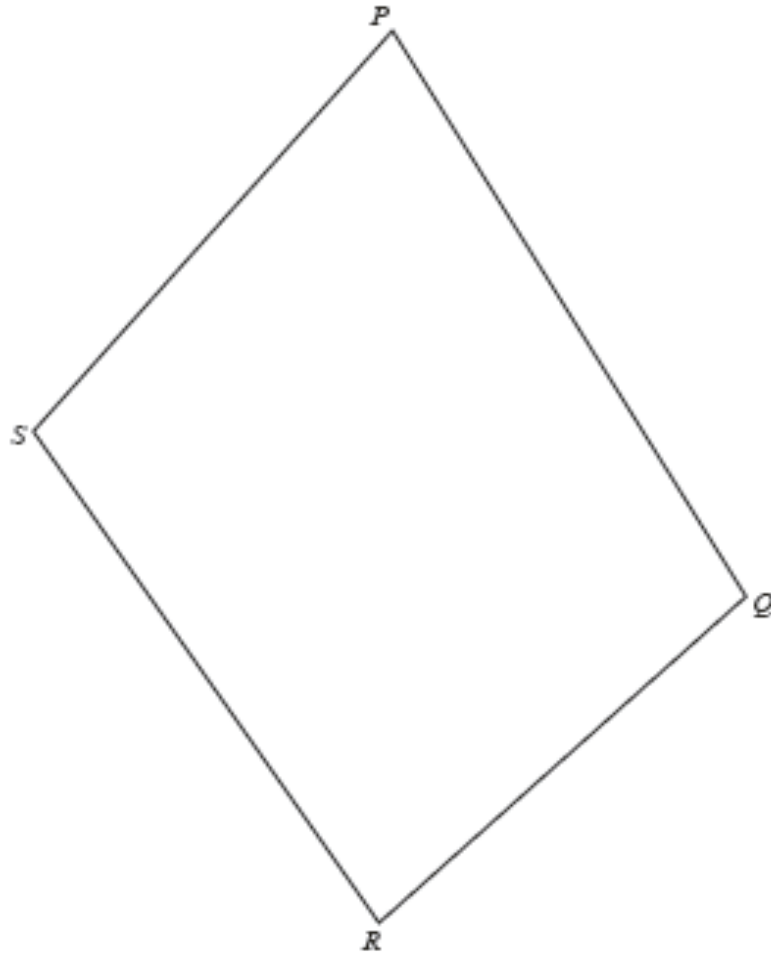
(iii) (a) Measure the shortest distance from  $B$  to  $AC$ .

Answer  $\dots\dots\dots$  cm [1]

(b) Work out the area of triangle  $ABC$ .

Answer  $\dots\dots\dots$  cm<sup>2</sup> [2]

(b)



This is an accurate diagram of quadrilateral  $PQRS$ .

- (i) Give a reason why it is not possible for  $P$ ,  $Q$ ,  $R$  and  $S$  to be points on the circumference of a circle.

*Answer* .....

..... [1]

- (ii)  $T$  is a point inside  $PQRS$  such that it is

- I more than 6 cm from  $R$
- II nearer to  $R$  than  $P$
- III nearer to  $PQ$  than  $QR$ .

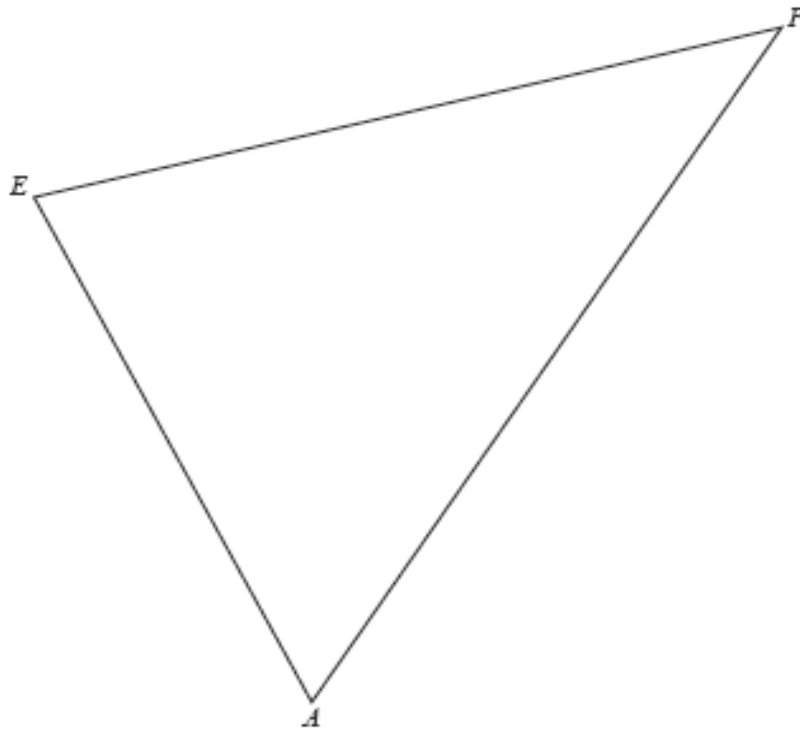
(a) Construct and shade the region within which  $T$  lies. [4]

(b) Find the maximum possible length of  $RT$ .

*Answer* ..... cm [1]

13

M/J17/10/22(b)



The diagram shows the positions of  $A$ ,  $E$  and  $F$ .

Construct and shade the region inside triangle  $AEF$  that is

- less than 6 cm from  $E$
- nearer to  $AF$  than to  $AE$
- nearer to  $A$  than to  $F$ .

[4]

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14 The diagram shows the quadrilateral  $ABCD$ . O/N16/11/20

(a) Measure  $\hat{DCB}$ .

*Answer*  $\hat{DCB} = \dots\dots\dots$  [1]

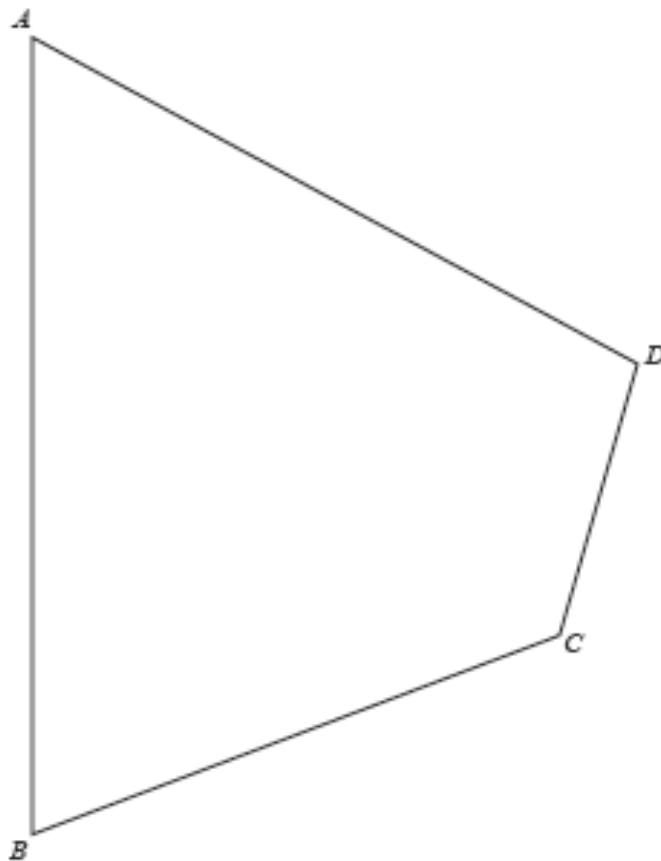
(b) (i) Construct the locus of points, inside the quadrilateral, that are 8 cm from  $B$ . [1]

(ii) Construct the locus of points, inside the quadrilateral, that are 5 cm from  $AB$ . [1]

(iii) These two loci meet at  $P$ .

Mark, and label, the point  $P$  on the diagram and measure  $PD$ .

*Answer*  $\dots\dots\dots$  cm [1]

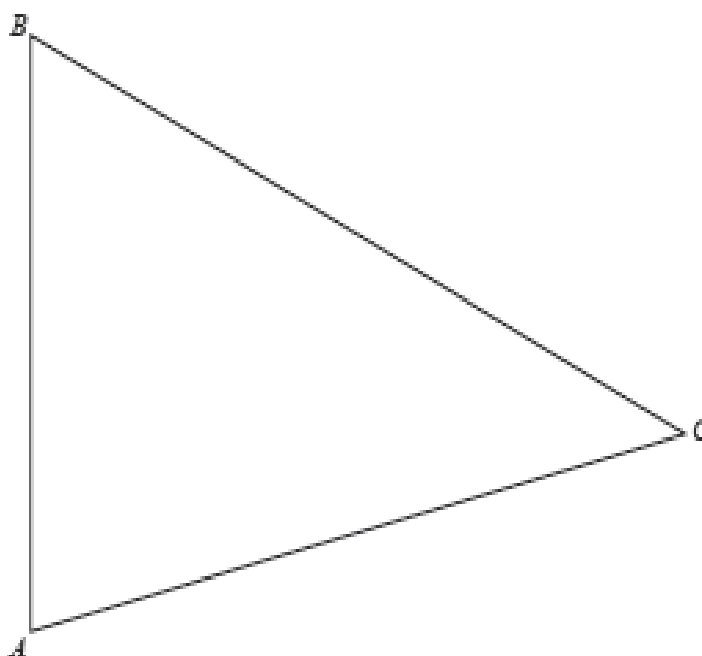


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15 The diagram shows triangle  $ABC$ .

ON/16/12/22

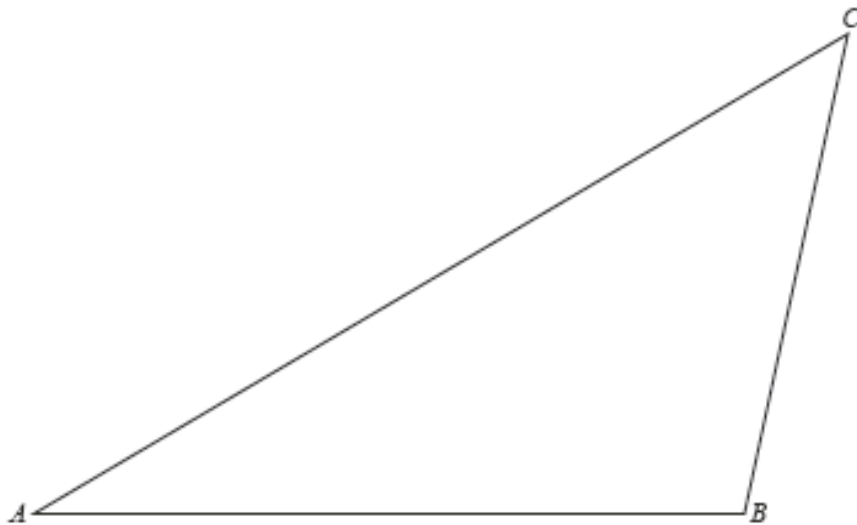
- (a) Construct the locus of points, inside triangle  $ABC$ , that are equidistant from  $A$  and  $B$ . [1]
- (b) Construct the locus of points, inside triangle  $ABC$ , that are equidistant from  $AB$  and  $BC$ . [1]
- (c) On the diagram, shade the region inside triangle  $ABC$  which contains the points that are nearer to  $A$  than to  $B$  and nearer to  $BC$  than  $AB$ . [1]



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16 The diagram shows triangle  $ABC$ .

M/J16/11/22



(a) Measure  $\hat{ABC}$ .

*Answer*  $\hat{ABC} = \dots\dots\dots$  [1]

(b) On the diagram, construct the locus of points, **inside** triangle  $ABC$ , that are

(i) 4 cm from  $B$ , [1]

(ii) 2 cm from  $AC$ . [1]

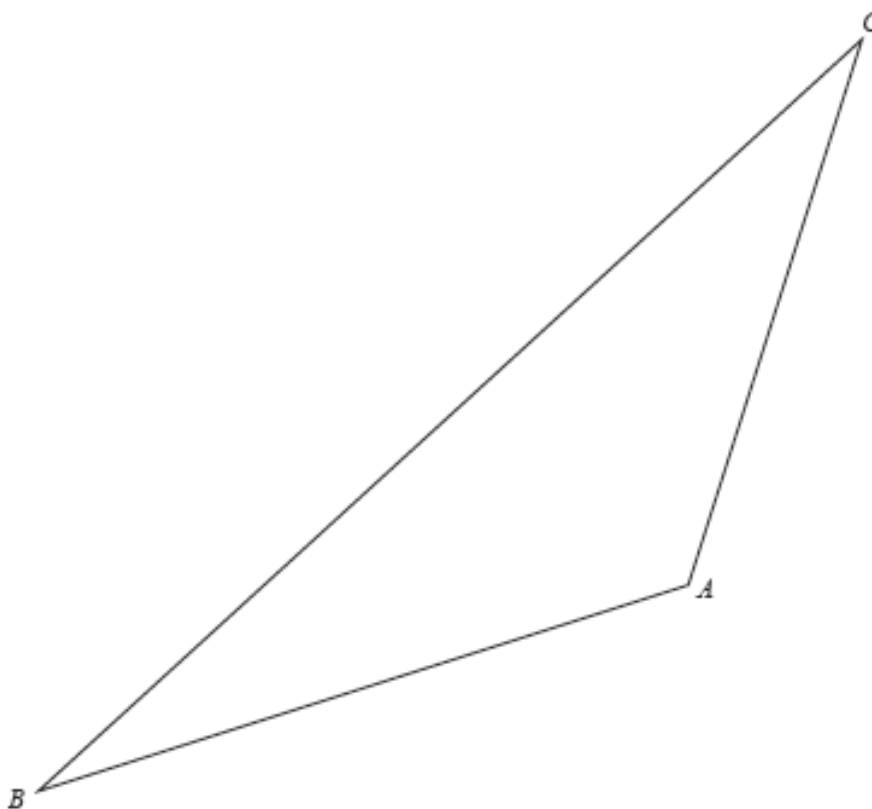
(c) The point  $P$  is  
4 cm from  $B$ ,  
2 cm from  $AC$ ,  
**and** nearer to  $A$  than to  $C$ .

Label the position of  $P$  on the diagram and find the length of  $AP$ .

*Answer*  $AP = \dots\dots\dots$  cm [1]

17 The diagram below shows triangle  $ABC$ .

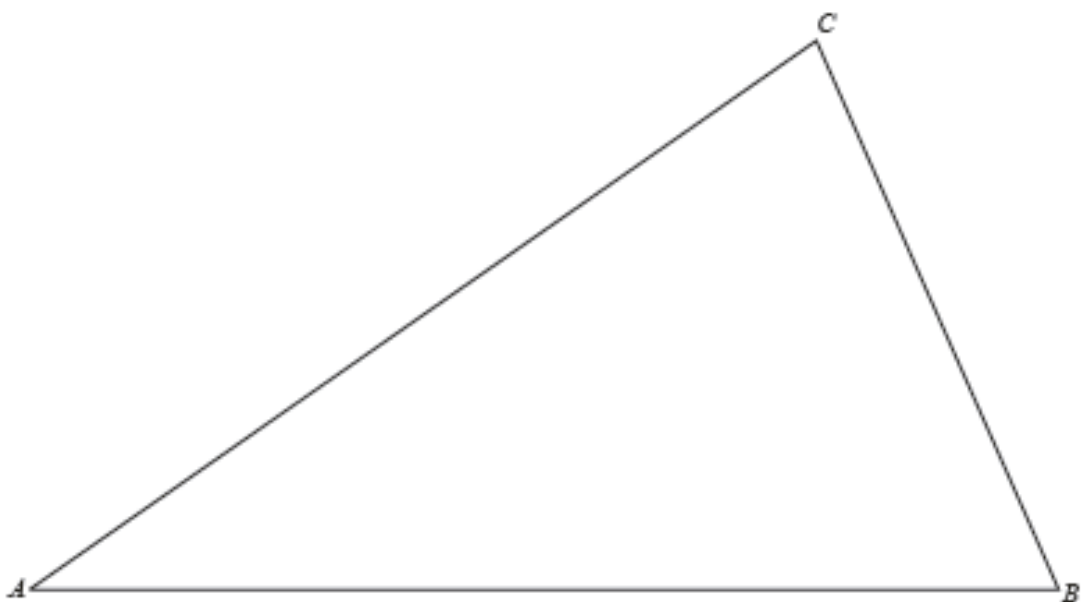
M/J16/12/12



- (a) On the diagram construct the locus of points inside the triangle that are
- (i) 3.5 cm from  $A$ , [1]
  - (ii) equidistant from  $AC$  and  $BC$ . [1]
- (b) On the diagram, shade the region inside the triangle containing the points that are more than 3.5 cm from  $A$  and closer to  $AC$  than to  $BC$ . [1]

- 18 In this question use a ruler and compasses only.  
Show all your construction arcs.

The diagram shows a triangular field  $ABC$ .  
The scale is 1 centimetre represents 50 metres.



Scale : 1 cm to 50m

- (a) Construct the locus of points that are equidistant from  $A$  and  $B$ . [2]
- (b) Construct the locus of points that are equidistant from the lines  $AB$  and  $AC$ . [2]
- (c) The two loci intersect at the point  $E$ .  
Construct the locus of points that are 250m from  $E$ . [2]
- (d) Shade any region inside the field  $ABC$  that is  
 \* more than 250m from  $E$   
 and  
 \* closer to  $AC$  than to  $AB$ . [2]



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In order to do well in this paper, candidates need to

- be familiar with the content of the entire syllabus
- be competent at basic arithmetic
- produce clear, accurate graphs and diagrams
- set out their work in clear, logical steps
- be able to select a suitable strategy to solve a mathematical problem.

### Marking scheme

1(a)(i)	Perpendicular bisector of $AB$ with correct arcs	2	<b>B1</b> for acceptable bisector with no/incorrect arcs
1(a)(ii)	Bisector of angle $ADC$ drawn with two pairs of correct arcs	2	<b>B1</b> for acceptable bisector with no/incorrect arcs
1(b)	Correct area shaded	1	<b>FT</b> <i>their</i> reasonable attempts at <b>(a)</b> and <b>(b)</b>

2	Acceptable bisector of $\angle ABC$ with correct arcs	2	<b>B1</b> for bisector with no / incorrect arcs
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3(a)(i)	Correct parallelogram with construction arcs at $B$ and $C$	3	<b>B2</b> for correct parallelogram without arcs <b>or</b> <b>B1</b> for two of $AB = 8$ or $DC = 8$ <b>or</b> $BD = 11$ or $BC = 6$
3(a)(ii)	101 to 105	1	<b>FT</b> <i>their</i> triangle $ABD$
3(a)(iii)	4[.0] to 4.4 with $AE$ drawn	1	<b>FT</b> <i>their</i> $AE$ from correct parallelogram

4(a)	$264^\circ$ to $268^\circ$ inclusive	1	
4(b)	Acceptable quadrilateral $ABCD$	1	
4(c)(i)	Acceptable perpendicular bisector of $AB$	1	
4(c)(ii)	Acceptable bisector of angle $ABC$	1	
4(d)	Correct region (top left-hand corner) shaded	1	<b>dep</b> on two reasonably accurate intersecting lines

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5(a)	$96^\circ$ to $98^\circ$	1	
5(b)(i)	Acceptable bisector of angle $ABC$ with correct arcs	2	<b>B1</b> for an acceptable angle $ABC$ bisector with no/incorrect arcs. or <b>SC1</b> for a very short line correctly constructed.
5(b)(ii)	Acceptable perpendicular bisector of $AB$ with two pairs of correct arcs	2	<b>B1</b> for an acceptable perpendicular bisector of $AB$ with no/incorrect/only one pair of arcs or <b>B1</b> for two pairs of correct arcs, without a line joining the points of intersection or <b>SC1</b> for a very short line correctly constructed.

6(a)	All four angle bisectors	2	<b>B1</b> for one, two or three angle bisectors
6(b)	Acceptable perpendicular bisector of $OQ$	1	

7	Correct region shaded	3	<b>B1</b> for line parallel to $AB$ 3 cm away for length of barn <b>B1</b> for 2 correct semicircles radius 3 cm centre A and B <b>B1</b> for region outside barn shaded between line parallel to $AB$ and attempt at two arcs centred A and B
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8(a)	Acceptable perpendicular bisector of $AB$	1	
8(b)(i)	Arc, centre $C$ , radius 7 cm	1	
8(b)(ii)	Bisector of angle $BAC$	1	
8(c)	$P_1$ and $P_2$ marked at intersections of <i>their</i> (a) with (b)(i) and (b)(ii)	1	dependent on correct types of loci in (b).

9(a)(i)	Correct construction with arcs	2	<b>B1</b> for correct triangle with arcs missing or arc 6 cm from $A$ or arc 9 cm from $B$
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9(a)(ii)	$77^\circ$ to $81^\circ$	1	FT <i>their</i> angle $BAC$
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10(a)	Acceptable quadrilateral with visible arcs	1	
10(b)(i)	Acceptable bisector of angle $ABC$	1	
10(b)(ii)	Acceptable perpendicular bisector of $BC$	1	
10(c)	Acceptable $PQ$ – dep. on correct types of loci in (b).	1	

11(a)	97 to 99 inclusive	1	
11(b)	Acceptable line	1	
11(c)	Full circle, centre $C$ , radius 5 cm	1	
11(d)	4.3 to 4.9 cm, dep. on two labelled intersections of an acceptable line and arc.	1	

12(a)(i)	$D$ correctly placed to the left of $AC$	2	<b>B1</b> for $DA = 9$ or $CD = 7$
12(a)(ii)	$44^\circ$ to $48^\circ$	1	FT
12(a)(iii)(a)	2.9 to 3.1	1	
12(a)(iii)(b)	19.1 to 20.8	2	<b>B1</b> for 13.2 to 13.4 seen
12(b)(i)	Opposite angles are both obtuse or both acute so their total is not 180 Or opposite angles are not supplementary	1	

12(b)(ii)(a)	Correct region shaded		<b>B1</b> for arc 6 cm from $R$ <b>B1</b> for angle bisector of $Q$  <b>B1</b> for perpendicular bisector of $PR$  After B2, <b>SC1</b> for 'correct' region shaded provided only slight inaccuracy with the other line/curve
12(b)(ii)(b)	7.9 to 8.3	1	FT

13	Correct region shaded	4	<b>B1</b> for arc 6 cm from $E$ <b>B1</b> for angle bisector of $EAF$  <b>B1</b> for perpendicular bisector of $AF$  After B2, <b>SC1</b> for 'correct' region shaded provided only slight inaccuracy with the other line/curve
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<b>14 (a)</b>	125° to 129°	1	
<b>(b) (i)</b>	correct arc	1	
<b>(ii)</b>	correct straight line	1	
<b>(iii)</b>	$PD = 3.4$ to $3.8$ cm WWW	1 dep	

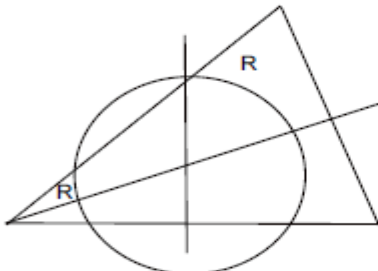
Dependent on correct types of loci, that intersect.

<b>15 (a)</b>	Perpendicular bisector of $AB$ .	1	
<b>(b)</b>	Bisector of angle $ABC$ .	1	
<b>(c)</b>	Correct (bottom right) region shaded.	1 ✓	

FT for two intersecting lines – slightly inaccurate but correct types of loci.

<b>16 (a)</b>	101° to 103°	1	
<b>(b) (i)</b>	Circular arc, centre $B$ , radius 4 cm.	1	
<b>(ii)</b>	Line parallel to $AC$ , 2 cm away.	1	
<b>(c)</b>	$AP = 6.2$ to $6.6$ cm	1	

<b>17 (a) (i)</b>	arc radius 3.5 cm, centre $A$	1	
<b>(ii)</b>	bisector of angle $ACB$	1	
<b>(b)</b>	Correct region shaded	1	

<b>18 (a)</b>	Correct perpendicular bisector of $AB$ with 2 pairs of correct arcs isw	<b>2</b>	<b>B1</b> for accurate with no/wrong arcs or <b>M1</b> for correct intersecting arcs with no or wrong line
<b>(b)</b>	Correct angle bisector at $A$ with two pairs of correct arcs isw	<b>2</b>	<b>B1</b> for accurate with no/wrong arcs or <b>M1</b> for two pairs of correct arcs with no or wrong line
<b>(c)</b>	Circle centre $E$ radius 5 cm isw	<b>2FT</b>	<b>FT</b> circle centre <i>their</i> $E$ radius 5 cm provided (a) and (b) attempted  <b>M1</b> for $250 \div 50$ oe soi e.g. from arc If 0 scored <b>SC1</b> for circle centre <i>their</i> $E$
<b>(d)</b>		<b>2</b>	<b>cao</b>  <b>B1</b> for each If 0 scored, <b>SC1</b> for two 'correct' regions but in part (c), centre correct but radius incorrect